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## ABSTRACT

This learning module reviews basic algebraic principles, the use of algebra for solving mathematical problems in the workplace as related to electronics, mechanics, computer operations, printing, and the general concepts of algebraic formulas. The course provides students with a practical knowledge of algebra in areas such as variable algebraic addition, subtraction, multiplication and division of monomials and multinomials, and solving for specified variables from given formulas. The module includes units for six class sessions. Each unit includes the following materials: rationale, learning objectives, curriculum notes and references for the instructor, course outline, introduction, evaluations, information sheets, problems to solve, and transparency masters, pretests and posttests, and course evaluations. (KC)

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# GENERAL WORKPLACE ALGEBRA

A Numerical Approach  
to Problem Solving

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**Colorado Community College and Occupational Education System**  
**United States Department of Education**  
**Corporate, Workforce, & Economic Development, a division of**  
**Pikes Peak Community College**  
**Current, Inc.**

# **GENERAL WORKPLACE ALGEBRA**

## **A Numerical Approach to Workplace Problem Solving**

The curriculum reviews basic algebraic principles, the use of algebra for solving mathematical problems in the workplace as related to electronics, mechanics, computer operators, and pressman, and the general concepts of algebraic formulas. This course provides students with a practical knowledge of algebra in areas such as variable algebraic addition, subtraction, multiplication and division of monomials and multinomials, and solving for specified variables from given formulas. It was designed to support occupational training needs in a work environment.

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## INTRODUCTION

The Workplace Classroom is a set of 11 curriculum modules created by workplace educators from Pikes Peak Community College in collaboration and partnership with employees of Current, Inc., a large greeting card company in Colorado Springs, Colorado. The partnership was formed through an 18-month federal workplace research and development grant from the United States Department of Education awarded to the Colorado Community College and Occupational Education System. Teachers in the project designed, developed and field-tested curricula and materials for the 11 basic skills courses through the process of identifying and understanding the culture of the workplace and the learning needs of the individuals working within it.

The Pikes Peak staff chose not to rely on ready-made materials or programmed texts with which to teach classes. Instead, teachers and curriculum specialists interviewed employees, created job profiles, developed customized assessments, and invited student participation in the development of class content. The result is a unique set of curriculum modules in learning to learn, reading, writing, communication, problem solving, English as a second language, math and algebra that reflect learning needs of real people in a large printing/manufacturing environment. These modules were designed as six week, two hour classes, but the learning rationale and intentions could easily be modified to accommodate longer or shorter sessions.

The idea of following a design process involving the active and continuous commitment and participation of the employee and the employer provides a fresh look at the development of curricula and instruction. The goal of this process is to develop a curriculum product that enhances the basic literacy skills of adults and increases critical thinking and problem solving skills that are easily transferred to occupational improvement. The Pikes Peak staff felt that the best way to reach this goal was to involve employees and employers in the many levels of curriculum development and design.

We believe that these curriculum products are genuine reflections of sound adult learning theory that says adults must have relevant learning experiences that build on prior knowledge and in some way advance positive change in their daily work lives. These modules were built through the active participation and assessment of the adult students for whom they were designed. Those of us who developed these products encourage other workplace educators to use them in part or as complete modules, keeping in mind that their very design welcomes the change and diversity that other workplace environments are sure to lend to them. We feel that the authenticity of our curricula will provide ideas and incentive to other teachers and curriculum specialists who are beginning new programs or are looking for ways of improving existing curricula.

Best of luck with any or all of the Workplace Literacy Modules.



Rita Moore, Project Leader

Workplace Literacy Grant Pikes Peak Community College

***"This course helped me to understand equations better and feel more at ease with the abstract thought of algebra".***

**--Algebra Student**

# **GENERAL WORKPLACE ALGEBRA**

## **A Numerical Approach to Workplace Problem Solving**

### **Rationale**

Adults who would take an algebra class have either never had algebra or feel they have forgotten enough to feel anxious. Many people who take algebra feel that they don't really understand it but go through the motions. These are often people who "drop out" of math in high school and so they never get the experience of understanding that they might have had if they had continued their math education. Many people anticipate failure at this math level.

It is important in a workplace algebra class that the content be familiar. Learners will commonly report a feeling of "I've had this before, but now I really understand it." It is also important that some of the methodology be different. If the learner is a concrete person, teaching some of the concepts the traditional way might reinforce the idea that math isn't for everyone. What may be necessary instead is to make the abstractions concrete at first before using the ideas in their traditional form. Manipulatives are an excellent tool for creating a concrete experience while minimizing anxiety for the students. Cuisenaire rods, two color chips, and algebra tiles are some of the newest manipulatives being used. They may be purchased at any teacher supply store or through educational catalogues. Seeing familiar content presented using a different methodology that addresses a concrete learning modality will encourage the student that while the subject is the same, they are different and there is every chance of success.

It is also important that the student feel that they are in control of the mathematics involved. There are lessons where the student creates equations of their own. Workplace applications are especially important here, since algebra of all the mathematics has the bad reputation of being nice but useless. It is important to make the subject relevant, both for the person and for the workplace environment.

*Rita Moore*  
Project Leader

*Nancy Wilson*  
Author

# **GENERAL WORKPLACE ALGEBRA: A Numerical Approach to Workplace Problem Solving**

## **SESSION I:**

Introduction, overview and paperwork  
Journal / Discuss: Previous if any Algebra experience  
Using variables and writing equations using rods  
Writing numbers in terms of others  
Function example: Number of squares on the edge of a square  
Number of squares on the border

## **SESSION II:**

Function example: Give the next number in the sequence  
Intro to equations  
Distributive Law with arithmetic and algebra

## **SESSION III:**

Substitution of values for variables  
Order of operations  
Balancing equations with inverse operations (2 step)  
Distributive law in equations

## **SESSION IV:**

Function: Measuring Circumference and Diameter, discovery of  
Pi. Create formula. Use to get diameter of ball.  
Positive and negative numbers: Introduction using hips.

## **SESSION V:**

Review positive and negative numbers  
Using reciprocals in solving equations

## **SESSION VI:**

Solving proportions  
Review equations and positive/negative numbers  
Post test and paperwork

# **GENERAL WORKPLACE ALGEBRA:**

## **A Numerical Approach to Workplace Problem Solving**

### **Session I**

*"This course helped to solve equations in another electronics course I am taking."*

Algebra Student

### **Learning Intentions:**

- Inventory of workplace algebra needs, overview of algebra as a whole subject, variables in the concrete using Cuisinaire rods, application of algebra, creating an equation, introduction to algebra.

### **Curriculum Notes:**

- Curriculum notes and references follow course outline.

### **Course Outline:**

#### **I. Administrative Details**

- A. Attendance and class roster
- B. Participant Data Sheets
- C. 4 x 6 cards
  - name
  - work extension
  - department name and number
  - work days and hours
  - home phone (optional)
  - personal information
- D. Participant learner packet
- E. Portfolio

#### **II. Purpose of class**

- Confidence and skill improvement.

#### **III. Write: How was Algebra for you before? Or if no Algebra, why not?**

- Share. Compare.



#### IV. Algebra: Arithmetic Revisited.

<u>Skills</u>	<u>Language</u>	<u>Functions (Formulas)</u>
• Equations solve	Symbolism	Number relationships
• + numbers	Translations	Writing formulas
• Laws of math	Vocabulary	

#### V. Cuisinaire Rods: Using variables

- Write equations for nine. Use colors.
- Variable: A symbol that stands for a quantity, usually a letter of the alphabet.
- Rods - use color to symbolize quantity.

$$4R + 1 = 9 \quad 3G = 9 \quad B + 1 = 9 \quad 9w = 9$$

- Solving equations means to uncover the unknown quantity.

#### VI. Number Trick

pick a number	8	
triple it	24	
add 8	32	
double the answer	64	$\frac{2(12x + 8) - 16}{6}$
subtract 16	48	
divide by 6	8	

#### VII. Do in terms of as many as possible in 1 minute then shortcut later.

- In terms of: A) write out B) shortcut

• In terms of 5 -	$19 = 5 + 5 + 5 + 4$ $12 = 5 + 5 + 2$ $+14 = 5 + 5 + 5 - 1$ $45 = 8(5) + 5$	or $3(5) + 4$ $2(5) + 2$ $3(5) - 1$ $8(5) + 5$
-------------------	--	--

• In terms of 7-	$19 =$ $12 =$ $+14 =$	in terms of 3-
------------------	-----------------------------	----------------

- In terms of whatever. 2 ways.

$$\begin{array}{r} 23 \\ 18 \\ +31 \\ \hline \end{array}$$

**VIII. Substitution:  $F = 5$  Substitute rest of examples**

$$19 = F + F + F + 4$$

$$12 = F + F + 2$$

$$14 = F + F + F - 1$$

$$45 = 8 F + 5$$

**IX. Creating function (formula)**

Edge      Border

**X. Homework**

**XI. Evaluations**

- Daily Journals

## Curriculum Notes:

### I. Administrative Details

**Pre-Evaluation:** Every program will have some kind of record-keeping process. The procedures we've listed have worked for us. Daily attendance sheets and class rosters are kept. Students who complete four out of the six classes receive a certificate of completion at the end of the course. Participant data sheets are federal forms that information that provides a profile of the company. Four by six cards are used by instructors to collect information that will enable them to get in touch with a student outside of class or work if necessary. The participant learner packet contains a summary brochure about the program; who they may call if they have a question about scheduling, class content, etc.; an explanation of the process for claiming classtime as work hours; a copy of an individual education plan, and a sample of the certificate they may receive upon class completion.

The portfolio is a folder with paper for journaling; daily evaluation sheets; and a place for students to collect their work for their own assessment and for the instructor's assessment of their work progress.

The pre-evaluation is really a form of self-assessment. Students are asked to list goals related to the course and assign numerical weight to them. At the end of the class the cards are re-examined for progress and students again assign numerical weights to their progress. (Please see attached assessment activity explanation). Students may also take a pre/post test. Administrative details at the end of the session.

Post evaluation is linked to the goal setting and assessment activity above and/or pre and post-evaluation instruments designed by teachers. The course evaluation (attached) and instructor evaluation (attached) are designed to guide the instructional team in making curriculum modifications as well as changes in teaching strategies.

### II. Algebra: Arithmetic Revisited

There are three main sections in the study of algebra: Skills such as solving equations, language and specific symbolism, and functions such as formulas.

### III. Cuisinaire Rods: Using variables

Cuisinaire rods are colored bars that are used to teach number theory to children. They can be used to teach number relationships such as quality and number magnitude, ration and proportion, and using symbolism to represent numbers. Ten different sized rods which also differ in color represent the numbers one through ten. Because the colors differ, referring to a "one" as white is comfortable, and it is only one step further to refer to it as "W".  $W=1$  is an equation that students readily understand and indeed are quit comfortable

### III. Cuisinaire Rods: Using variables continued

contributing more complex ones such as  $4R+1=9$ . Have students write down as many equations they can that would represent 9. In this way, variables and equation writing are very concrete and comfortable. When the goal becomes to recover the unknown value and students learn equation solving skills, the instructor can still refer back to this experience with the rods. It is a very powerful tool.

### VI. Number Tricks

Number tricks are a source of amazement to students, even simple ones that the instructor can create. The number tricks throughout the course are intended to catch the attention of the learners, demonstrate some algebra that they haven't yet studied in the course as an overview, and are basically fun. For this trick, each student should privately choose a number and write it down without sharing. Then the instructor should ask them to 1) triple it, then 2) add 8, then 3) double the answer, then 4) subtract 16, then 5) divide by 6, finally giving the answer as each person's original number.

It is interesting to demonstrate the trick using both numbers and algebraic symbolism at the same time:

#### ARITHMETIC

Original number:	8
Triple it:	24
Add 8:	32
Double:	64
Subtract 16:	48
Divide by 6:	8

#### ALGEBRA

x
3 x
3 x + 8
2 ( 3 x + 8 )
2 ( 3 x + 8 ) - 16
[ 2 ( 3 x + 8 ) - 16 ] / 6

Show how to simplify the algebra. It is ok for some people not to be able to follow. Stress that knowing all of these skills is a course goal and everyone will be able to by its end.

$$[ 2 ( 3 x + 8 ) - 16 ] / 6 = ( 6 x + 16 - 16 ) / 6 = 6 x / 6 = x.$$

Through the algebra, what is hidden becomes apparent; that is, the operations undo each other and the original number is left. This is difficult to see using a specific number example but it will work no matter what the number chosen since the expression is actually an identity.

They can now do addition problems in terms of other numbers. The best one to begin with is using the numbers in terms of 10, since this one is the one we use when adding anyway. The point here is to stress the addition of like terms.

## VI. Number Tricks Continued

$$19 = 1 ( 10 ) + 9$$

$$12 = 1 ( 10 ) + 2$$

$$+14 = 1 ( 10 ) + 4$$

$$\hline 45 = 3 ( 10 ) + 15$$

They can then do the same problem in terms of a different number, say 5:

$$19 = 3 ( 5 ) + 4$$

$$12 = 2 ( 5 ) + 2$$

$$14 = 2 ( 5 ) + 4$$

Point out that the answer is correct, but that it looks different from the first one.

$$\hline 45 = 7 ( 5 ) + 10$$

Subtraction can also be used to obtain a correct equality.

$$19 = 3 ( 5 ) + 4$$

$$12 = 2 ( 5 ) + 2$$

$$14 = 3 ( 5 ) - 1$$

Negative numbers can result here...

$$\hline 45 = 8 ( 5 ) + 5$$

Have them do this same problem 3 different ways, using 3 different "in terms of " numbers and verifying the correct equality at the end.

## VII. In terms of

"In terms of" is a powerful mathematical and scientific concept that is presented here at its most elementary level : using addition. I introduce it by saying that like people are known in terms of others (someone else's wife, brother, mother, or son) numbers can also be known in terms of others. This concept is a great lead in to equations and variables and the student will be choosing the numbers and the variable to be used. This small choice is very empowering for the student.

Choose a number, say 11. Demonstrate how 11 can be shown as an arithmetic problem involving repeated addition and a remainder:

$$11 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 \quad ( \text{ in terms of } 1 )$$

$$11 = 2 + 2 + 2 + 2 + 2 + 1 \quad ( \text{ in terms of } 2 )$$

$$11 = 3 + 3 + 3 + 2 \quad ( \text{ in terms of } 3 )$$

$$11 = 4 + 4 + 3 \quad ( \text{ in terms of } 4 )$$

$$11 = 5 + 5 + 1 \quad ( \text{ in terms of } 5 )$$



## VII. In terms of Continued

Show another example, say 19:

$$19 = 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 1$$

$$19 = 3 + 3 + 3 + 3 + 3 + 3 + 1$$

$$19 = 4 + 4 + 4 + 4 + 3$$

$$19 = 5 + 5 + 5 + 4$$

$$19 = 6 + 6 + 6 + 1$$

$$19 = 7 + 7 + 5$$

It is easy for students to follow the symbolic short cut to show the repeated addition: multiplication. They immediately prefer it.

$$19 = 9 ( 2 ) + 1$$

$$19 = 6 ( 3 ) + 1$$

$$19 = 4 ( 4 ) + 3$$

$$19 = 3 ( 5 ) + 4$$

$$19 = 3 ( 6 ) + 1$$

$$19 = 2 ( 7 ) + 5$$

## VIII. Substitution

Next the students are going to use substitution. They choose a symbol for the "in terms of" number and make a statement such as Let  $F = 5$ . Point out the similarity of the arithmetic and the algebra, stressing that algebra is only arithmetic using symbolism.

Arithmetic

$$19 = 3 ( 5 ) + 4$$

$$12 = 2 ( 5 ) + 2$$

$$14 = 3 ( 5 ) - 1$$

$$\underline{45 = 8 ( 5 ) + 5}$$

Algebra Let  $F = 5$

$$19 = 3 F + 4$$

$$12 = 2 F + 2$$

$$14 = 3 F - 1$$

$$\underline{45 = 8 F + 5}$$

Stress that 5's get added to other 5's as F's get added to other F's. Like terms is easy to teach at this time.

Have them practice a couple of these giving recognition for the creativity they have recently developed in writing their own algebra.

## IX. Creating Function

This is one example of a function. It allows students to see the power and beauty of the development of a formula. They will write their own based on the information given and use it to make predictions about problems that would be inconvenient to model. It begins with the concrete and moves to the abstract. Not all students will see the pattern at first, so it is helpful to have a discussion about what people see.

Draw a series of rectangles like those here. Have them create a chart of the results or do one on the board or overhead and use it with the class.

### SQUARES



### CHART:

Edge	Border
2	4
3	8
4	12

Continue creating a few more squares that can be easily drawn. Then it is interesting to ask what the number would be if that on the edge was 12? Or 100? This will lead the students into the abstraction of the formula. Have them explain how they arrived at answers for these without drawing them. Hopefully most or all of the class can see that the number of squares around the border equals 4 times those on one edge minus 4. Have them explain why this is so. Different people may view this differently and so their explanations will differ. The important thing is for people to define the relationship here in their own minds.

Symbolize the formula having students contribute the variables to use:

$B = 4E - 4$ . Some will choose to view it as  $B = 4(E - 1)$ .

## X. Homework

Homework: Have students do this problem or one similar "in terms of" three different numbers:

$$\begin{array}{r} 34 \\ 17 \\ + 23 \\ \hline 74 \end{array}$$

Class: \_\_\_\_\_  
Date: \_\_\_\_\_  
Instructor: \_\_\_\_\_

	Name	Dept. Number	S. S. Number
1.	_____	_____	_____
2.	_____	_____	_____
3.	_____	_____	_____
4.	_____	_____	_____
5.	_____	_____	_____
6.	_____	_____	_____
7.	_____	_____	_____
8.	_____	_____	_____
9.	_____	_____	_____
10.	_____	_____	_____
11.	_____	_____	_____
12.	_____	_____	_____
13.	_____	_____	_____
14.	_____	_____	_____
15.	_____	_____	_____
16.	_____	_____	_____
17.	_____	_____	_____
18.	_____	_____	_____
19.	_____	_____	_____
20.	_____	_____	_____
21.	_____	_____	_____
22.	_____	_____	_____
23.	_____	_____	_____
24.	_____	_____	_____
25.	_____	_____	_____
26.	_____	_____	_____
27.	_____	_____	_____
28.	_____	_____	_____
29.	_____	_____	_____
30.	_____	_____	_____
31.	_____	_____	_____

# SKILLS FOR A COMPETITIVE WORKFORCE PARTICIPANT DATA SHEET

Please fill out the following information. Print or write clearly. This information will be used for demographic and statistical purposes only.

## SECTION I (Identification)

Name: \_\_\_\_\_ Social Security Number: \_\_\_\_\_  
Last Name, First Name Middle Initial

Street Address: \_\_\_\_\_ City: \_\_\_\_\_ Zip Code: \_\_\_\_\_

Phone Number: (\_\_\_\_) \_\_\_\_\_ - \_\_\_\_\_

Department: \_\_\_\_\_ Position: \_\_\_\_\_

## SECTION II (demographic information)

1. Yrs. with company (circle one): a. unemployed b. 0-5 c. 6-10 d. 11-15 e. over 16

2. Age: \_\_\_\_ 3. Sex: M F

4. Ethnicity (circle one): a. White b. Black c. Hispanic d. American Indian/Alaska Native  
e. Asian/Pacific Islander f. Other

5. Single: Y N 6. Is English your second language? Y N  
Head of Household: Y N

7. Participating in (circle one or more):  
a. Basic Skills Program  
b. GED Program  
c. ESL Program

## SECTION III (outcome information)

Assesment Planning:

8. Course Title: \_\_\_\_\_ (check one: \_\_\_\_ Basic Skills, \_\_\_\_ GED, \_\_\_\_ ESL)

	Goals	Assessment Tool	Pre-Asses Results	Post-Asses Results	Improved
9.	Test Higher on Basic Skills: _____				Y N
10.	Improved Communication Skills: _____				Y N
11.	Increased Productivity: _____				Y N
12.	Improved Work Attendance: _____				Y N
13.	Increased Self-Esteem: _____				Y N

14. Contact Hours: \_\_\_\_ - \_\_\_\_

15. Course Title: \_\_\_\_\_ (check one: ☐ Basic Skills. ☐ GED. ☐ ESL)

	Goals	Assessment Tool	Pre-Asses Results	Post-Asses Results	Improved
16.	Test Higher on Basic Skills: _____				Y N
17.	Improved Communication Skills:				Y N
18.	Increased Productivity:				Y N
19.	Improved Work Attendance:				Y N
20.	Increased Self-Esteem:				Y N

21. Contact Hours: \_\_\_\_\_

22. Course Title: \_\_\_\_\_ (check one: ☐ Basic Skills. ☐ GED. ☐ ESL)

	Goals	Assessment Tool	Pre-Asses Results	Post-Asses Results	Improved
23.	Test Higher on Basic Skills: _____				Y N
24.	Improved Communication Skills:				Y N
25.	Increased Productivity:				Y N
26.	Improved Work Attendance:				Y N
27.	Increased Self-Esteem:				Y N

28. Contact Hours: \_\_\_\_\_

29. Course Title: \_\_\_\_\_ (check one: ☐ Basic Skills. ☐ GED. ☐ ESL)

	Goals	Assessment Tool	Pre-Asses Results	Post-Asses Results	Improved
30.	Test Higher on Basic Skills: _____				Y N
31.	Improved Communication Skills:				Y N
32.	Increased Productivity:				Y N
33.	Improved Work Attendance:				Y N
34.	Increased Self-Esteem:				Y N

35. Contact Hours: \_\_\_\_\_



# STUDENT EVALUATION

## Pre-Evaluation

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Course: \_\_\_\_\_ Instructor: \_\_\_\_\_

GOALS	1 20%	2 40%	3 60%	4 80%	5 100%
I need to improve my communication skills.					
I need to improve my productivity.					
I need to improve my work attendance.					
I need to improve my self-esteem.					

List 4 goals related to the following that you want to improve in:

Communications: \_\_\_\_\_

Reading: \_\_\_\_\_

Writing: \_\_\_\_\_

Math: \_\_\_\_\_

Rate yourself on a scale of 1-5 as to where you are with these goals. 1 would be the lowest and 5 would be the highest.

## EVALUATION

### STUDENT DAILY LOG

NAME: .

DATE:

CLASS:

1. What did you learn today? What did you find useful about the lesson? How was it interesting?
  
  
  
  
  
  
  
  
  
  
2. What did you find not necessarily useful, and what could have been done to improve the effectiveness of the lesson?
  
  
  
  
  
  
  
  
  
  
3. What other reactions do you have to the class, materials, discussion, etc.?
  
  
  
  
  
  
  
  
  
  
4. Are you comfortable with the material? Why or why not?
  
  
  
  
  
  
  
  
  
  
5. How have you used any of the information learned in previous classes?

# GENERAL WORKPLACE ALGEBRA: A Numerical Approach to Workplace Problem Solving

## Session II

*"This course has helped me to think of problems in a different way - looking for patterns."*

Algebra Student

### Learning Intentions:

- Review Session 1 building equations using pattern blocks, introduction to elementary equation solving, pretest assessment.

### Curriculum Notes:

- Curriculum notes and references follow course outline.

### Course Outline:

#### I. Warm up: In terms of

$$4x + 2 = 22$$

$$4(6) + 2 = 26$$

#### II. Number Trick:

Pick number.

Add 7.

Double result.

Subtract 4

Divide result by 2

Subtract original number

Answer = 5

$$\frac{2(x + 7)}{2} - 4 - x = \frac{2x + 14}{2} - 4 - x = \frac{2x + 10}{2} - x = x + 5 - x = 5$$

#### III. Function: Give function.

From pattern. Give next in pattern. Discuss

$$4x + 2$$

$$16 - x$$

$$x^2$$

### III. Function: Give function. Continued:

- Fraction blocks to make equation

Left hand side = Right hand side. (agree on symbols)  
H, T, R, A = Triangle  
A) L.H.S. = R.H.S. Write other equation using blocks.  
 $2 T = 1 H$   $3 R = 1 H$   
or  
 $1 H = 3 T$   $6 A = 1 H$

B) Can we say combinations  
 $3A + 1 R = H$

C) Multiples  $4R = 2 H$  or  $3T + A = 1 H +$   
fill in the blank

- Use 14 blocks.
- Make equations
- Write it (them)

### IV. Technique of Opposites

- Add and subtract from both sides - tech. of opposites.  
 $H + A = 2 T + A$   $T = 3A$   $6A = 2 T$   
 $T - A = 2A$   $6A + T = 3 T$

### V. Assessment - Algebra PreTest

### VI. Evaluation

- Daily Journal  
Students record comments about the class and how they can use their new skills on the job.

## Curriculum Notes:

### I. Warm Up:

Go over homework and follow up with a similar problem. Each person could do this problem in terms of any number of their choice, and should do it using arithmetic and algebra.

### II. Number Trick:

See the session 1 notes if necessary. When discussing it with the whole class, show both arithmetic and algebra.

#### Arithmetic

Choose a number: 6  
Add 7. 13  
Double the result 26  
Subtract 4. 22  
Divide by 2 11  
Subtract  
Original: 5

#### Algebra

$X$   
 $X + 7$   
 $2(X + 7)$   
 $2(X + 7) - 4$   
 $[2(X + 7) - 4] / 2$   
 $[2(X + 7) - 4] / 2 - X$

The answer, no matter what the original number, is 5.

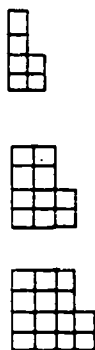
To show why this is so, simplify the algebra:

$$[2(X+7) - 4] / 2 - X = [2X + 14 - 4] / 2 - X = [2X + 10] / 2 - X = X + 5 - X = 5$$

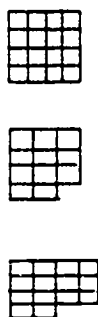
### III. Function

Function example: Draw these patterns on the board and have students see if they can guess a relationship for each one. They don't have to symbolize this, although some will want to. It is perfectly all right to discuss the patterns people see in english. On example #1 There is always one new row of squares, then always 2 extra.

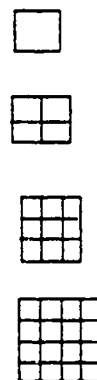
A)



B)



C)





### III. Function Continued:

Have students discuss any formulas that are used on this work site and show how to manipulate them. Students may need some examples using substitution with some of these formulas.

$$A.) 4x + 2$$

$$B.) 16 - x$$

$$C.) x \times 2$$

This is another way, similar to the rods, to create equations with concrete objects. Stress that equations have a Left hand side and a Right hand side. Agree on symbols for the pattern blocks, which come in different geometric shapes, with each shape a different color.

Hexagon:H

Trapezoid:T

Rhombus:R

Triangle:A

Have them create a series of equations, statements of equality, about the relationships they see in the shapes. Symbolize the relationships.

$$2T = 1H$$

$$1H = 3R$$

$$3A + 1R = H$$

$$1H = 2T$$

$$1H = 6A$$

Then have them use multiples and fill in the blanks:  $4R = 2H$   
or  $3T + A = 1H + ?$

Finally have students create an equation using at least 14 blocks. The equation should include both a left and a right hand side. They should write their equation in symbols. Compare these.

Students that are concrete learners, who have to learn by doing, will enjoy this activity. Likewise, students who are used to the abstract level, or who prefer to learn by observing will go crazy hating this activity.

### IV. Technique of opposites

Use a basic equation with the pattern blocks like:

$$H + A = 2T + A$$

Show that an "A" can be subtracted from both sides. The resulting equation will be different, but it will still be a statement of equality:

$$H = 2T$$

#### **IV. Techniques of opposites continued**

Try  $6A = 2T$ . Then add  $T$  to both sides. Stress that this must be done to both sides in order to balance the equation.

$6A + T = 3T$  is still a true statement, although it is different than the original.

#### **V. Assessment - Pretest**

Pretest. This pretest may or may not cover the same quantity of content as the course. It is a goal. The post test should only cover content of the course, so these assessments do not match in content.

# GENERAL WORKPLACE ALGEBRA: A Numerical Approach to Workplace Problem Solving

## Session II

### Pretest

Name:

Multiply these using the distributive law as in the example:

Example:  $8(16) = 8(10 + 6) = 8(10) + 8(6) = 80 + 48 = 128$

1.  $5(42) =$

2.  $12(104) =$

3.  $6(X + 4) =$

4.  $-2(X - 10) =$

Find the answers:

5.  $-6 - (-2) =$

6.  $-5 + 17 =$

7.  $15 - (7) =$

8.  $-8 + 22 =$

9.  $-6 + (-4) =$

10.  $-6 - 9 =$

11.  $(-5)(6) =$

12.  $(-9)(-3) =$

13.  $30 - 10 =$

if  $A = 3$ ,  $B = 4$ ,  $X = 0$  and  $Y = 2$ , evaluate these:

14.  $ABY =$

15.  $XB - YA =$

16.  $\frac{A + X}{B} - Y =$

# GENERAL WORKPLACE ALGEBRA: A Numerical Approach to Workplace Problem Solving

## Session III

*"After taking this course I can now look at equations (formulas) and know how to go about solving them, step by step."*

Algebra Student

### Learning Intentions:

- Symbolism in algebra, substitutions and order of operations, equality and equation solving using inverse operations, distributive law.

### Curriculum Notes:

- Curriculum notes and references follow the course outline

### Course Outline:

#### I. Number Trick

#, +3, double, -6, /2 = #

#, +3, double, -6, -number = #

#### II. Assessments - Review

#### III. Order of Operations; Language;

Substitution A=6, B=4, C=12, D=5, E=0, F=1

A) A+B    B) 2A - C    C) 4B + 3C    D) 2(A+B)

E) AF - D

F) FAB    G) ED + EF    H) A(B+C)    I) C(A-D)

#### IV. Equality + Balancing Equation

$$5 = 5$$

$$10 = 2(5)$$

$$7 = 10-3$$

$$10 = 20/2$$

$$5+2 = 5+2 \quad 10+4 = 2(5)+4 \quad 7-2 = 10-3-2 \quad 4 \times 10 = 20/2 \times 4$$

Try: Write New Equality (balancing)

$$5 = 5$$

$$6 = 4+2$$

$$10 = 10$$

$$20 = 20$$

use x

use +

use -

use /

## V. To solve for unknowns:

$$6x = 42 \quad x + 3 = 8 \quad x - 5 = 12 \quad x/2 = 7$$

Explain in words first (describe) then write with language. Get answers first for  $x$ , then show with steps inverse operations.

Operations: Use opposite (inverse) op.  $x$  is  $/$ ,  $+$  is  $-$ ,  $-$  is  $+$ ,  $/$  is  $x$ .  
 $6x/6$

Find side with  $x$ . 2) Decide what operation. 3) Balance.  
 $6x = 42$

Practice using steps on one-step eq.

$$\text{A.) } x + 8 = 24 \quad \text{B.) } x - 7 = 19 \quad \text{C.) } 3x = 51 \quad \text{D.) } x/4 = 8$$

## VI. Two-step equation

Reverse order of ops:  $x = 5$      $2x - 6 =$   
Recover 5     $2x - 6 = 4$

Practice:

$$4x + 2 = 14 \quad 3x - 5 = 10 \quad x/2 + 5 = 10 \quad x/3 - 2 = 6$$

## VII. Distributive law

Multiply left to right

$$\begin{array}{cccccc} 15 & 14 & 15 & 13 & 12 & 17 \\ \underline{x5} & \underline{x6} & \underline{x7} & \underline{x8} & \underline{x9} & \underline{x4} \end{array}$$

Arithmetic

$$\begin{aligned} \text{Algebra: } 5(15) &= 5(10+5) = 5(10) + 5(5) \\ 5(15) &= 5(8+7) = 5(8) + 5(7) \end{aligned}$$

Algebra

$$\begin{aligned} x &= 10 \\ 5(x+5) &= 5x + 5(5) \\ x &= 8 \\ 5(x+7) &= 5x + 5(7) \end{aligned}$$

$$\text{Add} \quad \text{Try } = 5(15) = 5(+ ) =$$

$$\text{Sub.} \quad 5(15) = 5(18-3) = 5(18) - 5(3)$$

$$5(x+5) = 75 \quad 5(x+7) = 75$$

$$5x + 25 = 75 \quad 5x + 35 = 75$$



### **VIII. Practice Solving**

$$6(x+2) = 42$$

$$4(x-8) = 16$$

$$7(x-1) = 63$$

### **IX. Handout Assignment**

### **X. Evaluation**

#### **A. Daily Journal**

Students record comments about the class and how they can use their new skills on the job.

# General Workplace Algebra: A Numerical Approach to Workplace Problem Solving

## Session III

### Handout Assignment

I. If  $A=2$ ,  $B=5$ ,  $C=0$ ,  $D=1$ ,  $E=8$ , and  $F=7$ , calculate:

1.  $AF - D$

2.  $ABD$

3.  $3E$

4.  $\frac{E}{A}$

5.  $\frac{F+E}{A}$

6.  $2(B+C)$

7.  $5(F-D)$

8.  $4(A+B)$

9.  $A(B+F)$

II. Solve these equations showing steps:

10.  $X + 5 = 24$

11.  $X - 9 = 12$

12.  $X + 6 = 27$

13.  $2X + 1 = 41$

14.  $3X - 2 = 13$

15.  $7X - 5 = 12$

III. Write in terms of 4. Let  $F = 4$

Arithmetic      Algebra

16. 26

$$\begin{array}{r} +47 \\ \hline 73 \end{array}$$

17. Solve each equation above for  $F$  using equation steps.

26 =

47 =

73 =

Solve:

18.  $3(x+2) = 27$

19.  $5(x-4) = 35$

20.  $7(x+2) = 245$

## Curriculum Notes:

### I. Number trick:

Arithmetic

Algebra

Choose a number:	11	$x$
Add 3	14	$x + 3$
Double:	28	$2(x+3)$
Subtract 6:	22	$2(x+3)-6$
Divide by 2:	11	$[2(x+3)-6]/2$

Simplify the algebra: By now, the students will be able to help, hopefully.  
 $[2(x+3)-6]/2 = [2x+6-6]/2 = 2x/2 = x$

### II. Review Assessments

### III. Language Practice and Order of Operations

Substitution: Let  $A = 6$ ,  $B = 4$ ,  $C = 12$ ,  $D = 5$ ,  $E = 0$ , and  $F = 1$ .

A.  $A+B =$

B.  $2A-C =$

(Stress  $2A$  means 2 times  $A$ )

C.  $4B+3C$

(Again 4 times  $B$  and 3 times  $C$ )

D.  $2(A+B)$

(Add first, then double)

E.  $AF - D$

(multiply first, then subtract)

F.  $FAB$

(multiply all three)

G.  $ED + EF$

(multiply each of the two pairs then add)

H.  $A(B+C)$

(add, then multiply)

I.  $C(A-D)$

(subtract, then multiply)

### IV. Equalities and Balancing the Equation

This is the abstraction of the work done in Session 2 with pattern blocks. If students seem to like the blocks, work with them at the same time. They may prefer to go directly to the abstract, and in this case, forget the blocks. Stress that the ideas are the same, however.

Start with a basic number truth:  $5 = 5$ . It is still a statement of equality, (but a different equality) if we add the same quantity to both sides:  $5+2 = 5+2$ . The same is true with  $10 = 2(5)$ . It is still a true statement to add 4 to both sides:  $10+4 = 2(5)+4$ . Likewise, we can subtract the same quantity from

#### IV. Equalities and Balancing the Equation Continued:

both sides and have a true statement as a result:  $7 = 10 - 3$  can become  $7 - 2 = 10 - 3 - 2$ . Adding and subtracting the same quantity from an equality results in another equality. We can also multiply and divide both sides by the same quantity:  $5 = 5$  might be where we start, but  $5(4) = 5(4)$  is also true.  $20 = 2(10)$  can become  $20/2 = 2(10)/2$  without losing the truth of the equality.

Have students practice altering the beginning true fact to obtain a second different, but also true fact using numbers. It should not take too many of these, but do enough examples so that students are very comfortable.

$5 = 5$ :  
(multiply both by any quantity)

$$5(3) = 5(3)$$

$6 = 4 + 2$ :  
(add the same quantity to both)

$$6 + 8 = 4 + 2 + 8$$

$$10 = 10: \quad 10 - 8 = 10 - 8$$

$$20 = 20: \quad \frac{20}{4} = \frac{20}{4}$$

(divide both by the same)

#### V. Solving for Unknowns

$6X = 42$  is a symbolic form of the statement "6 times something equals 42." We have the answer already, and X represents the number that when multiplied by 6, the result is 42. Have students give the missing number, 7, before the step using the inverse operation. The formal algebra process is to analyze the  $6x$  part.  $6X$  means 6 times X. The opposite operation of multiplication (its inverse) is division. If both sides of this equation were divided by 6, two things would happen: first, the new statement of equality would still be an equality, since 6 was used on both sides. Secondly, the equation would be simpler, since  $6x/6 = x$ . The division by 6 would "undo" the multiplication leaving  $1X$  or  $x$ . Therefore, the unknown value is 7, since  $42/6$  is 7.

It should be shown symbolically also:  $6X = 42$ ,  $6x/6 = 42/6$ ,  $x = 7$

- 1) Find the side with the variable,
- 2) Analyze what operation is being used
- 3) Use the inverse of the operation with the same quantity on both sides,
- 4) Simplify.

Operation	Inverse
+	-
-	+
x	/
/	x

## V. Solving for Unknowns Continued

Examples:

$$X + 8 = 24$$

Operation: +

Inverse: -

$$\text{Subtract 8 from both sides: } x + 8 - 8 = 24 - 8$$

$$\text{Simplify: } X = 16$$

$$X - 7 = 19$$

Operation: -

Inverse: +

$$\text{Add 7 to both sides: } X - 7 + 7 = 19 + 7$$

$$\text{Simplify: } X = 26$$

$$3x = 81$$

Operation: x

Inverse: /

$$\text{Divide both sides by 3: } 3x/3 = 81/3$$

$$\text{Simplify: } x = 27$$

$$x/4 = 8$$

Operation: /

Inverse: x

$$\text{Multiply both sides by 4: } x/4 \times 4 = 8 \times 4$$

$$\text{Simplify: } x = 32$$

## VI. Two-step equations

Review substitution with order of operations from session 2:

$$x = 5. \quad 2X - 6 = ?$$

Multiply first, then subtract. The answer is 4.

The equation form of this with the goal of recovering the 5 is:

$$2x - 6 = 4.$$

In solving the equation, we undo it in reverse order from the substitution.

Step 1: The equation says to subtract 6, so the inverse is to add 6 to both sides.

$$2x - 6 + 6 = 4 + 6 \text{ and simplify: } 2x = 10$$

Step 2: The equation says that x is multiplied by 2, so the inverse would be to divide both sides by 2.  $2x/2 = 10/2$  and simplify:  $x = 5$ .

## VI. Two Step Equations

It helps students to see some problems worked both directions, because they know what the goal is and can see the parallel between steps. Again, one method is arithmetic, and the other is algebra and they are very close in how they work. Algebra is only an extension of arithmetic.

Give students some practice in the two-step method:

$$4X+2 = 14$$

$$3x-5 = 10$$

$$x/2+5 = 10$$

$$x/3-2 = 6$$

## VII. The Distributive Law

Multiplication from left to right is one way to model the distributive law in a way that also empowers the student. Many of these can be easily done mentally working from left to right.

$$15$$

$$x5$$

---

$$10 \times 5 + 5 \times 5 = 50 + 25 = 75$$

$$14$$

$$x6$$

---

$$10 \times 6 + 4 \times 6 = 60 + 24 = 84$$

Try some mental multiplication:

$$15$$

$$x7$$

---

$$13$$

$$x8$$

---

$$12$$

$$x9$$

---

$$17$$

$$x4$$

---

The next step is to write the problem horizontally, then let  $x = 10$  and symbolize the problem algebraically. Then try it with  $x = 8$ . Then try it with subtraction.

$$15 \times 5 = 5 \times 15 = 5(10+5) = 5(10) + 5(5) = 50 + 25 = 75$$

$$\text{let } x = 10. 5(10+5) = 5(x+5) = 5x + 5(5) = 5x + 25$$

$$5 \times 15 = 5(8+7) = 5(8) + 5(7) = 40 + 35 = 75$$

$$\text{let } x = 8. 5(8+7) = 5(x+7) = 5x + 5(7) = 5x + 35$$

$$5 \times 15 = 5(18-3) = 5(18) - 5(3) = 90 - 15 = 75$$

$$\text{let } x = 18. 5(18-3) = 5(x-3) = 5x - 5(3) = 5x - 15$$

The main point is that both the numbers inside the parentheses must be multiplied by the one on the outside and then if they can be simplified later, fine.

## VII. The Distributive Law Continued

Equation form. Have them solve the equations, starting with the distributive law.

$$5(x+5) = 75$$

$$5x + 25 = 75$$

$$5x + 25 - 25 = 75 - 25$$

$$5x = 50$$

$$5x/5 = 50/5$$

$$x = 10$$

$$5(x+7) = 75$$

$$5x + 35 = 75$$

$$5x + 35 - 35 = 75 - 35$$

$$5x = 40$$

$$5x/5 = 40/5$$

$$x = 8$$

## VIII. Practice Solving

$$6(x+2) = 42$$

$$4(x-8) = 16$$

$$7(x-1) = 63$$

## IX. Handout for Homework

# GENERAL WORKPLACE ALGEBRA: A Numerical Approach to Workplace Problem Solving

## Session IV

*"This course has certainly stimulated my thinking, thereby helping me to reason more logically when confronted with repairs of an electrical nature."*

Algebra Student

### Learning Intentions:

- Review, Discovery of Pi and circumference formula with application, addition and subtraction of integers.

### Curriculum Notes:

- Curriculum notes and references follow the course outline.

### Course Outline:

#### I. Work goal

#### II. Number trick:

# Add 4, x 4, -4, /2. Subtract orig. #. Subtract 6.

#### III. Review Substitution

A=5, B=3, C=2

1.  $AB+C =$

2.  $A(B+C) =$

#### IV. Go Over Handout

#### V. Circumference and Integers

- Function. Measuring circumference and diameter
- Discover pi. Create formula  $C = \pi \times D$
- Use to get diameter of ball.
- I Numbers. Red = -1    Black = +1     $R+B = 0$   
Practice. Dice.



# GENERAL WORKPLACE ALGEBRA: A Numerical Approach to Workplace Problem Solving

## Session IV

### Handout Assignment

Add.

1.  $(-6) + (-9) =$

2.  $(+4) + (+8) =$

3.  $(-5) + (-10) =$

4.  $(-8) + (+5) =$

5.  $(-7) + (-10) =$

6.  $(+4) + (-10) =$

7.  $(-3) + (-2) =$

8.  $(-4) + 7 =$

9.  $3 + (-8) =$

10.  $(-2) + (-5) + (-4) =$

11.  $7 + (-7) =$

12.  $-5 + 6 =$

Subtract.

13.  $(-5) - (-2) =$

14.  $(-3) - (-1) =$

15.  $(+4) - (+6) =$

16.  $(+5) - (+10) =$

17.  $(+8) - (-8) =$

18.  $(+8) - (+8) =$

19.  $5 - (-2) =$

20.  $-4 - (-2) =$

21.  $-4 - (+2) =$

22.  $-3 - (+10) =$

23.  $-3 - 12 =$

24.  $5 - (-8) =$

Solve. Show steps.

1.  $3X + 2 = 8$

2.  $5X - 1 = 11$

3.  $13 = 3X - 2$

4.  $41 = 8X + 1$

5.  $2(X+3) = 16$

6.  $4(X-3) = 28$

7.  $18 = 6(X+2)$

8.  $20 = 5(X-1)$

9.  $X/2 + 4 = 10$

10.  $13 = X/5 - 2$

## Curriculum Notes:

### General Session Notes:

Supplies: 4 or 5 string segments of different lengths, a selection cylinders such as cans or jars, a few rulers or yardsticks, a calculator and a ball such as a tennis ball or a basketball. These supplies will be used to discover and calculate pi and apply the circumference formula.

2 color chips like those sold in teacher supply stores or a set of about 15 each of black and red (or any other 2 colors) poker chips. These are good manipulatives to teach the addition and subtraction of integers.

### II. Number Trick

Arithmetic

Algebra

Choose a number:	7	X
Add 4:	11	$X + 4$
Multiply by 4:	44	$4(x+4)$
Subtract 4:	40	$4(x+4) - 4$
Divide by 2:	20	$[4(x+4) - 4]/2$
Subtract original:	13	$[4(x+4) - 4]/2 - x$
Subtract 6:	7	$[4(x+4) - 4]/2 - x - 6$

Simplify the algebra:  $[4x+16-4]/2-x-6 = [4x+12]/2-x-6 = 2x+6-x-6 = x$   
Students might well be able to simplify independently by now.

### III. Review Substitution

A=5, B=3, C=2. Calculate  $AB+C =$  and  $A(B+C) =$

### IV. Go over the handout

### V. Circumference.

Start by having students measure the circumference and diameter of the cylinders and record the results for everyone to see. Measure the circumferences by wrapping the string around the object then measuring the length of string with the ruler or yardstick. When all the measurements have been taken, have students calculate the quotient of the circumference divided by the diameter. In all cases, the quotient is approximately 3.14, which mathematicians refer to as pi.

Circumference, diameter, and pi are always in this relationship, and symbolize this for students:  $C/D = \pi$ . Manipulate the formula also for them by multiplying both sides by D:  $C/D \times D = \pi \times D$  or  $C = \pi \times D$ . Also since  $C = \pi \times D$  and both sides are divided by pi, then we have  $C/\pi = D$ .

## V. Circumference Continued

To summarize, there are now three forms of the function we have discovered:

$$C/D = \pi$$

$$C = \pi \times D$$

$$C/\pi = D$$

Now it is possible to calculate the diameter of the ball we have without cutting it in half.  $C/\pi = D$  is the best form of the function to use, so we measure the circumference, divide it by pi, and that gives us its diameter.

Have students practice using the three forms of the formula by substituting values for either C or D and calculating the other.

## Adding and Subtracting Integers

Introduce this section by discussing adult applications of negative and positive numbers: income and credit cards. When we go into debt, we have a negative number, and the goal is to add positive values to the debt until we bring the balance up to zero.

Many people get confused with their experience history in integers, particularly with subtraction, so again it is helpful to demonstrate the concept in a new, concrete way. This method is a fun, nonthreatening way to show both addition and subtraction.

To begin, define one of the colors as positive and the other as negative. For this explanation, we will agree that black is positive and red is negative. A one to one correspondence of the two colors will equal zero, ie., 1 black + 1 red = 0 or 3 black + 3 red = 0. Give practice with this because zero will play a large part in the operations to come.

Define addition as adding colored chips to the workspace or wallet, and work with it alone for awhile before moving to subtraction. It also would be helpful not to go symbolic too early, but have the learners concentrate on the modeling first before introduce the ideas in written form. Stress the money connection in several examples using "income" and "debt" or "credit". Demonstrate a few problems with chips such as these below:

$(+3) + (+2)$  means to start with three black chips in the workspace and then add two black chips for a total of 5 black chips. (or +5)

$(-3) + (-1)$  means to start with three red chips and then add one more red chip for a total of 4 red chips. (or -4)

### **Adding and Subtracting Integers Continued**

Modeling integers of different signs begins the same way with the added step of simplifying the answer: If any zeros can be removed, do this step. For example:  $(+3) + (-2)$  means to start with 3 black chips and then add in two red chips. There are two sets of zeros ( $2 \text{ blacks} + 2 \text{ reds} = 0$ ), so remove these from the workspace. There remains only 1 black chip, so the answer is  $+1$ .

$(-4) + (+7)$  means to start with 4 red chips in the workspace and then add in 7 black chips. Removing the 4 zeros leaves only 3 black chips left, so the answer is  $+3$ .

After a few examples such as these, begin to symbolize these for the students as you give examples for them to model. At least 8 examples are necessary for them to begin to see the pattern in the concrete, even though some of them will remember the algebraic rules.

Subtraction is defined as removing chips from the workspace. Instead of removing zeros to simplify however, zeros will be added. Give some zero practice if necessary:

Example:  $+3 + 0 + 0 = 3 \text{ blacks} + (1 \text{ black} + 1 \text{ red}) + (1 \text{ black} + 1 \text{ red})$   
 $+3$  can be modeled with 5 blacks + 2 reds.

$-5 + 0 + 0 + 0 = 5 \text{ reds} + (1 \text{ black} + 1 \text{ red}) + (1 \text{ black} + 1 \text{ red}) + (1 \text{ black} + 1 \text{ red})$   
 $-5$  can be modeled with 8 reds and 3 blacks.

$(-3) - (-2)$  means to start with 3 reds and then remove 2 reds from the workspace. There are enough reds to remove leaving only one left. The answer is  $-1$ .

$(-3) - (-4)$  means to start with 3 reds and then to remove 4 reds. There are not enough, so bring in one zero,  $(1 \text{ black} + 1 \text{ red})$ .  $-3$  can be modeled as 4 reds and 1 black. Remove the 4 reds and the 1 black remains, so the answer is  $+1$ .

$(+6) - (+2)$  means to start with 6 blacks and then to remove 2 blacks. There are enough to remove, so remove the 2 blacks, leaving only 4, so the answer is  $+4$ .

$(+6) - (+8)$  means to start with 6 blacks and then to remove 8 blacks. There aren't enough to remove 8, so pull in two zeros.  $+6$  can be modeled as 8 blacks and 2 reds. Remove the 8 blacks, leaving the 2 reds, so the answer is  $-2$ .

$(+4) - (-2)$  means to start with 4 blacks and then to remove 2 reds. There aren't any reds, so add in two zeros for a total of 6 blacks and two reds. Remove the 2 reds, leaving 6 blacks, so the answer is  $+6$ .

## Adding and Subtracting Integers Continued

$(-3) - (+7)$  means to start with 3 reds and then to remove 7 blacks. Add in 7 zeros, for a total of 10 reds and 7 blacks. Remove the 7 blacks leaving the 10 reds, so the answer is -10.

Some students will begin to take shortcuts by not adding the zeros, but by calculating the answer mentally. Great! This is the goal anyway, to go to the abstract. When they have some practice, it would be helpful to summarize the patterns, saying income for positive and debt for negative.

Get rich

Get poor

Add positives

Subtract positives

Subtract negatives

Add negatives

Finally, give some problems in written form, allowing modeling if they want, using the same language as with the modeling such as:

$(+3) + (-6) = -3$  because if you start with \$3 and want to spend 6, you have a debt of \$3.

$(-8) - (-3) = -5$  because if you have a debt of \$8 and \$3 of that debt is removed, then you are only \$5 in debt.

Give several written independent practice problems. Drop the + sign only after you explain that you mean it to be positive and that the + is understood.

# GENERAL WORKPLACE ALGEBRA: A Numerical Approach to Workplace Problem Solving

## Session V

*"This course provided me with a better understanding of algebra in formulating percentages and formulas."*

Algebra Student

### Learning Intentions:

- Review, using reciprocals in solving equations, combining like terms on the same side of equations.

### Curriculum Notes:

- Curriculum notes and references follow course outline.

### Course Outline:

#### I. Number Trick

- Use your house number. Double it. Add 5. x 50. Add age. Add 365 days in year. Subtract 615. Put in decimal for \$ and cents. \$ = house #, cents = age

Algebra:  $50(2x + 5) + A + 365 - 615$

$100x + 250 + A + 365 - 615$

$100x + A$  demo with #'s to show dollars and cents.

#### II. Adding and Subtracting Integers

- + numbers with dice, etc. Review, add and subtract.

#### III. Reciprocals in Solving Equations ( $\frac{2}{3}$ & $\frac{3}{2}$ ) = 1

$$\frac{3}{5}x = 12$$

$$\frac{4}{5}x + 1 = 13$$

$$\frac{3}{4}x = 9$$

$$\frac{1}{2}x - 3 = 2$$

$$\frac{2}{3}x = 6$$

$$\frac{9}{10}x - 2 = 18$$

### III. Reciprocals Continued

- Combining x's

A) in terms of 5  $F = 5$

$$32 = 6(5) + 2 \quad 6F + 2$$

$$7 = 1(5) + 2 \quad F + 2$$

$$19 = 3(5) + 4 \quad 3F + 4$$

$$\begin{array}{r} \hline 58 = 10(5) + 8 \quad 10F + 8 \end{array}$$

- Notice that x's add with other x's and that x and 1x are the same.

A)  $3x + 8x = 22$       B)  $7x - 2x = 45$       C)  $3x + 2x + 1 = 16$

D)  $9(x + 2) - 3x = 30$     E)  $5(x-4) + 2x = 50$

### IV. Handout - homework finish equation page. Do second page.

## Curriculum Notes:

### I. Number trick (from Hallmark on a birthday card!)

Arithmetic		Algebra
Use your house number	15	X
Double it:	30	2X
Add 5:	35	2X + 5
Multiply by 50:	1750	50(2x+5)
Add your age:	1789	50(2x+5) + A
Add 365 (for days in a year)	2154	50(2x+5) + A + 365
Subtract 615	1539	50(2x+5) + A + 365 - 615
Put in decimal for \$ and cents in a year	\$15.39	[50(2x+5) + A + 365 - 615]/100
Equals your house number and age		

Simplify:  $[50(2x+5) + A - 250]/100 = [100x + 250 + A - 250]/100$   
 $= [100x+A]/100 = X + a/100$  (the cents part)

### II. Adding and Subtracting Integers

Give lots of written examples of Adding and Subtracting integers. Keep the numbers easy, less than 30. Make sure people are comfortable.

### III. Reciprocals

Define reciprocals as that quantity that when multiplied by the number the answer is 1.  $3/5 \times 5/3 = 1$ , so  $5/3$  is the reciprocal of  $3/5$  and vice versa. Remind them that we can multiply both sides of an equation by any quantity. Show some examples on one step equations involving fractions:

$$3/5x = 12$$

Multiply b  $5/3$  on both sides:  $3/5 (5/3)x = 12 (5/3)$

Simplify  $1x = 20$

It may be necessary to review multiplication of fractions.



# **GENERAL WORKPLACE ALGEBRA: A Numerical Approach to Workplace Problem Solving**

## **Session V**

### **Handout Assignment - Equations**

Skill	Technique of opposites and balancing
Summary:	Distributive Law
	Reciprocals
	Adding like terms

**Solve the equations using steps.**

1.  $3X = 24$

2.  $3X + 1 = 13$

3.  $3X - 5 = 10$

4.  $5X - 8 = 17$

5.  $3(X+7) = 36$

6.  $4(X-8) = 20$

7.  $1X/2 = 8$

8.  $2X/3 = 10$

9.  $5X/8 = 15$

10.  $6X/7 = 30$

11.  $3X/4 + 5 = 17$

12.  $2X/3 - 7 = 9$

13.  $2(X+6)/3 = 10$

14.  $3(X-12)/4 = 9$

15.  $2X + 6X = 24$

16.  $5X - X = 32$

## Handout - Equations

### Page 2

Solve the equations using steps.

17.  $9(X+3) + 2X = 82$

18.  $4(X-2) - 2X = 6$

19.  $1X/2 + 3X/4 = 15$

20.  $2X/3 + 2X/3 + 8 =$

# GENERAL WORKPLACE ALGEBRA: A Numerical Approach to Workplace Problem Solving

## Session VI

*"This course helped relieve my algebra anxiety and helped to ease the pressure of learning because of the way this course was taught."*

Algebra Student

### Learning Intentions:

- Review, solving proportions using the cross product, applications of the distributive law, post test.

### Curriculum Notes:

- Curriculum notes and references follow course outline.

### Course Outline:

#### I. Number Trick

- Take any 2 digit number except 99. (more interesting if digits are different). Double. Add 4. Mult by 5. Add 12. Mult by 10. Subtract 320. Cross out zeros. = original.

$$5(2x+4) = 10[10x + 20 + 12] = 10(10x + 32) = 100x + 320 - 320 = 100/100x = x$$

#### II. Handout Key

- Discuss

#### III. Proportion Equation Introduction

$$2/3x = 6 \quad 5 \quad 1/2x = 22$$

- Examples: cross product:

$$2/3 = x/9 \quad 4/10 = x/15 \quad 3/6 = x/4$$

- Practice:

$$4/6 = x/9 \quad 8/10 = x/15 \quad 14/16 = x/24 \quad 2x/3 = 9/10$$

#### **IV. Review Positive/Negative Numbers**

$$\begin{array}{llll} -4 - 8 = & 5 - (-3) = & -8 - (-2) = & -6 + -12 = \\ -6 - 12 = & -7 - (-2) = & 6 - 12 = & 7 - 10 = \\ -6 - (-12) = & -6 + 12 = & & \end{array}$$

#### **V. Review Equations**

$$\begin{array}{lll} 3x = 18 & x + 9 = 12 & 2(x-8) = 20 \\ 5(x+2) + 3x = 26 & 7x - 4 - 2x = 11 & \end{array}$$

#### **VI. Post test**

#### **VII. Evaluation**

##### **A. Daily Journal**

Students record comments about the class and how they can use their new skills on the job.

#### **VIII. Administrative Details**

##### **A. Post-Evaluation**

##### **B. Course Evaluation**

##### **C. Instructor Evaluation**

## Curriculum Notes:

### I. Number Trick:

Take any 2 digit number except 99. The problem is more interesting if the digits are different:

Arithmetic

Algebra

35

X

Double it: 70

2X

Add 4 74

2X + 4

Multiply by 5 370

5(2x+4)

Add 12 382

5(2x+4) + 12

Multiply by 10 3820

10[5(2x+4)+12]

Subtract 320 3500

{10[5(2x+4)+12]-320}/100

{10[5(2x+4)+12]-320}/100

Simplify:  $\{10[5(2x+4)+12]-320\}/100 = \{10[10x+20+12] - 320\}/100$   
 $= \{10[10x+32]-320\}/100 = \{100x + 320 - 320\}/100$   
 $= 100x/100 = x$

### II. Review Handout

### III. Proportion Equation

Solving proportions in equations using the cross product short cut.

First review solving equations having fractions by using the reciprocal:

$\frac{2}{3} X = 6$  Multiply both sides by  $\frac{3}{2}$ :

$\frac{2}{3} (\frac{3}{2}) x = 6(\frac{3}{2})$

$1x = 9$

Then define a proportion. It helps to use workplace applications here, or examples from map reading or doubling the recipe when cooking. Proportions are a special type of equation. Each side of the equation has only one term and both are fractions with X as a missing numerator or denominator. An example might be:

$$\frac{2}{3} = \frac{x}{9}$$

One way to solve this might be to multiply both sides twice, once by each denominator:

### III. Proportion Equation Continued:

Multiply by 3      $\frac{2}{3}(3) = \frac{x}{9}(3)$

Simplify          $2 = \frac{3x}{9}$

Multiply by 9      $2(9) = \frac{3x}{9}(9)$

Simplify          $18 = 3x$

Divide             $6 = x$

A short cut is to use a cross product. Multiply the numerator of one side by the denominator of the other, and write the two product with an equal sign between them:

$$\frac{2}{3} = \frac{x}{9}$$

$$2(9) = 3x$$

$$18 = 3x$$

$$\frac{18}{3} = x$$

$$6 = x$$

Several examples will help clarify this.

# STUDENT EVALUATION

## Post-Evaluation

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Course: \_\_\_\_\_ Instructor: \_\_\_\_\_

GOALS	1 20%	2 40%	3 60%	4 80%	5 100%
I improved my goal in communication.					
I improved my goal in productivity.					
I improved my goal to increase my work attendance.					
I improved my goal to increase my self-esteem.					

**WORKPLACE LEARNING PROGRAM  
INSTRUCTOR EVALUATION  
CURRENT AND PIKES PEAK COMMUNITY COLLEGE**

Please check one response to each question.	E x c e l l e n t	V e r y G o o d	S a t i s f a c t o r y	N e e d s I m p r o v e m e n t	P o o r
1. The instructor is organized in his/her teaching of this class.					
2. The instructor projects warmth, friendliness and enthusiasm in his/her presentation.					
3. The instructor returns tests and assignments within one class session.					
4. The instructor encourages student participation in class.					
5. The instructor reacts in a positive manner to students' questions and responses.					
6. The instructor is willing to give individual help when you request it.					
7. The instructor clearly communicates how the course is related to your learning needs.					
8. The instructor is skilled and knowledgeable in the material.					
9. You feel comfortable with asking your instructor to teach what you feel is important to your learning needs.					
10. By reviewing your portfolio, you are familiar with the changes in your own learning.					

What comments do you have that will help in the design of future courses? \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



**WORKPLACE LEARNING PROGRAM  
PARTICIPANT EVALUATION  
CURRENT AND PIKES PEAK COMMUNITY COLLEGE**

Please answer the questions which follow. Your responses will help us in making improvements in the course.

1. How would you rate the content of this course?

Too Difficult		Just Right		Too Easy
5	4	3	2	1

2. How would you rate the quality of the instruction materials?

Very Interesting		Somewhat Interesting		Uninteresting
5	4	3	2	1

3. How useful was the course in helping you on the job?

Very Useful		Somewhat Useful		Not Useful
5	4	3	2	1

4. Overall, how satisfied were you with the course?

Very Satisfied		Somewhat Satisfied		Very Dissatisfied
5	4	3	2	1

5. What did you like the best about this course? \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

6. What could have been done to improve the effectiveness of the course? \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

7. How would you rate the quality of the instructional materials? \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

8. Would you like additional time spent on this subject? Yes \_\_\_\_\_  
No \_\_\_\_\_ If yes, what specific subjects? \_\_\_\_\_  
\_\_\_\_\_

9. In what specific ways has this course helped you to do your job better? \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

10. How has this course helped meet goals you set before taking it?  
\_\_\_\_\_  
\_\_\_\_\_
11. Would you recommend this course to a co-worker? Yes \_\_\_\_\_ No \_\_\_\_\_  
Why or Why not? \_\_\_\_\_  
\_\_\_\_\_
12. Do you feel more confident about your learning abilities because  
of this class? \_\_\_\_\_  
\_\_\_\_\_
13. Will what you learned in class make a positive, noticeable  
difference in your outside interests? \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

# General Workplace Algebra: A Numerical Approach to Workplace Problem Solving

## Session VI

### Post Test

Name: \_\_\_\_\_

Multiply these using the distributive law as in the example:

Example:  $8(16) = 8(10+6) = 8(10) + 8(6) = 80 + 48 = 128$

1.  $5(28) =$

2.  $7(109) =$

3.  $6(X+4) =$

4.  $2(X-10) =$

Find the answers:

5.  $-8 - (-5) =$

6.  $-4 + 12 =$

7.  $11 - (-5) =$

8.  $-2 + 9 =$

9.  $-4 + (-3) =$

10.  $-6 - 10 =$

11.  $(-5)(4) =$

12.  $(-8)(-2) =$

13.  $3 - 10 =$

If  $A=2$ ,  $B=6$ ,  $X=1$  and  $Y=3$ , evaluate these:

14.  $ABY =$

15.  $YA - YB =$

16.  $(A+X+Y)/B =$

Solve these equations:

17.  $3X + 4 = 10$

18.  $2(X+4) + 5X = 15$

19.  $3/7 = 6/X$

20.  $3(X+4) - 8 = 19$

## Post Test

### Page Two

21.  $2X/3 = 8$

22.  $[1(2X+8)]/2 = 15$

23-24. Do this problem in terms of a number of your choice. Use both Arithmetic and Algebra.

Arithmetic

Algebra

$$\begin{array}{r} 34 \\ 15 \\ +27 \\ \hline 76 \end{array}$$

25. Do this number trick using a number first then show why it works using algebra.

Choose a number. Add 3. Double the result. Subtract 6. Subtract the original number. The result is the number you started with.

Arithmetic

Algebra

Solve these equations:

20.  $3X+1 = 10$

21.  $2(X+4) + 5X = 25$

22.  $2/5 = 9/X$

23.  $X+4 = 2X + 6$

24.  $-2X - 6 = -10$

25.  $[-5X+3]/3 + 3 = 15$

Write an equation and then solve it.

26. A number increased by 6 then doubled is 5 more than 3 times the number. Find the number.

27. 6 less than a number is 5 more than twice the number. Find the number.